Evaluation of a novel space filling curve and its application to the P2P overlay network

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Abstract - P2P networks can achieve high scalability since they distribute service contents/resources to multiple nodes in the network. In a P2P network, it is necessary to search the resource location on the network when we use some contents/resources. Space filling curve is known as technique to map information of a multi-dimensional space such as location information onto a one-dimensional space such as ID. We have proposed a novel space filling curve for P2P overlay networks considering the proximity of nodes based on the geographical information of the nodes and reducing the communication delay proportional to the geographical distance of the nodes. By using the proposed space filling curve, we can convert geographic information of nodes into their ID (label) of P2P network. In this paper, we evaluate the space filling curve for P2P networks and apply it on a virtual network map. On the virtual network map, each node has the coordinated position information which is proportional to the communication delay with its neighbor nodes. Through the numerical evaluation, we confirmed that the proposed curve is more suitable for handling hierarchical-spread nodes than the conventional curves on the virtual network map.

Keywords: P2P, Overlay Network, Space Filling Curve, Virtual Network Map, Node Labeling

1 Introduction

In recent years, with the development of the communication technology, mobile terminals such as cellular telephones or the note PCs come to be connected frequently to the network with wireless communication. As a result, the user of mobile terminals become possible to receive e-mails or the multimedia contents such as the image, and movie, etc. without depending at the specific location and time. Moreover, a huge amount of terminals such as sensors deployed in the living space or terminals installed by cars are expected to compose the network in coming ubiquitous computing society.

It is difficult to keep a scalability to offer high quality service for such large amount of terminals because the load to the server concentrates in server-client type system mainly used in current computer network. Therefore, the Peer-to-Peer (P2P) network system attracts attention. In the P2P network system, each terminal manages data autonomously and distributedly, and communicates directly with other terminals. Gnutella[2] and BitTorrent[1] are typical P2P network technologies. P2P network can provide network services to millions of terminals by distributing the load of the terminals and network unlike the server-client type architecture.

On the other hand, in the ubiquitous computing society, it becomes available a location-aware service that selects appropriate services by using user's geographic information, and a contents-aware service that selects appropriate services considering of the state of sensors in the neighborhood. Current cellular phone terminals and many of in-vehicle navigation systems are equipped with GPS (Global Positioning System) and can easily acquire the geographical location information of each user.

Space filling curve is known as technique to map information of a multi-dimensional space such as location information onto the one-dimensional space such as ID. Location-aware services can be provided easily by using location information as a node ID on P2P networks. However, there is a trade-off between the inefficiency of space filling in simple method such as Z-Ordering (Lebesgue curve)[10] and high calculation cost of a curve with good filling efficiency.

We have proposed a novel space filling curve for P2P overlay networks considering the proximity of nodes based on the geographical information of the nodes and reducing the communication delay proportional to the geographical distance of the nodes [8]. The proposed method gives each node a label (ID) from the space coordinate of the node based on its geographic information and the link delay between the nodes. The proposed curve can let the space coordinate be converted into the one-dimensional space efficiently. In this paper, we evaluate the space filling curve for P2P networks and apply it on a virtual network map by Vivaldi[4]. On the virtual network map, each node has the coordinated position information which is proportional to the communication delay with its neighbor nodes.

The rest of this paper is organized as follows. In Section 2, we briefly introduce related work of construction methods of P2P networks. Next, we describe our space filling curve and its application to a virtual network map in Section 3. Then we present neumerical simulation results in Section 4. Finally, we conclude this paper in Section 5.

2 Related Work

In Peer-to-Peer(P2P) technology, each peer constructs application layer overlay network, searches contents or resources over this network by direct communication between peers without a server, and distributes and shares the resources. Gnutella[2] and BitTorrent[1] are typical technologies using the non-structured overlay networks. On the other hand, the structured overlay networks can search a resource on the P2P networks, and many technologies are proposed in recent years.

Especially, there are many technologies that use Distributed Hash Table(DHT). Chord[13], CAN (content addressable network)[11], Pastry[12], and Tapestry[15] are known well.

On the other hand, in order to enable the range search which treats the consecutive quantity, some techniques which do not use DHT for decentralization of data and search queries have been proposed. SkipNet[5] is one method for configuring a structured P2P overlay network. In this method, the range search is enabled by using SkipGraph[3] that uses consecutive values for ID instead of DHT that uses a hash function. SkipGraph achieves the search efficiency of the logarithmic order by hierarchically grouping one-dimensional node array using the balanced tree structure. However, the range specification search in SkipNet might need to search wider range than other methods, because SkipNet constructs a hierarchical overlay network by using single search key, without regard to the geographical location and multiple search keys. LL-net[6] is structured P2P overlay network where the area on the map is hierarchically divided into four sub areas, and in each hierarchy the overlay links should be the different length links. In LL-net, the scalability of system is lacked so that the existence of a special node that manages each area may be assumed, and the management cost of the overlay network may increase. Moreover, if the distribution of the node is not uniform, it might become impossible to converse from geographic coordinates into node ID easily, the hierarchical structure might become biased, and the search efficiency might turn worse. In Mill[9], the range search is enabled by connecting nodes with the ring structure by applying ID which based on geographical location information. Hereby, the search efficiency can be realized $O(logN)$ without special nodes.

Space filling curve[14] (Peano curve) is known as a technique to convert information on multi-dimensional space such as location information into information on one-dimensional space like ID. The typical space filling curves are Lebesgue curve (Z-Ordering, Fig.1), Hilbert curve (Fig.2), Sierpinski curve (H-indexing), and $\beta\Omega$-indexing, etc.

Vivaldi[4] is a technique to calculate the coordinate system autonomously by considering both the proximity of the physical network and real communication delay on a P2P network. This technique leads virtual coordinate system by gradually correcting the difference between Euclidean distance and the actual measurement delay between nodes using the spring model after each participation node decides its virtual coordinates autonomously.

3 Proposed Method

In this paper, we research a new configuration method for structured P2P overlay network considering delay variations between nodes. To match coordinates in the two-dimensional plane with delay variations of nodes we use the technique of Vivaldi when the coordinates are calculated.

The search efficiency can be improved when given an ID which has small distance from IDs of the physically neighboring nodes. Therefore, it can be good for labeling a node ID to use geographical information of nodes. The space filling curve is often used for this case. We have proposed a space filling curve in [8] with following features that (1) it is easily convertible on node ID on the P2P network from the geographic information, (2) information search that specifies the range is efficiently executable, and (3) it is easy to construct a hierarchical structure to give the scalability.

In this paper, we describe the method to let the proposed curve fit the delay variations of nodes.

3.1 Conventional Space Filling Curves

Some space filling curves have been proposed so far. Hilbert

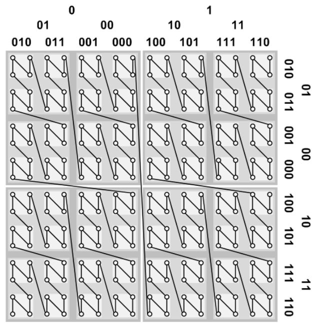

Figure 1: Lebesgue Curve (Z-Ordering)

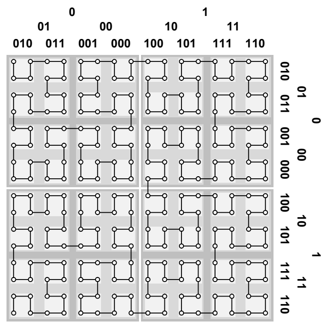

Figure 2: Hilbert Curve

curve, Sierpinski curve, and $\beta\Omega$-indexing fill the space so that the neighbor nodes on the two-dimensional space is converted to the close ID on the one-dimensional space. In these curves, the communication delay between logical neighbor nodes is suppressed small, and the range search can be searched efficiently. The performance of the Lebesgue curve is not better than that of the above curves, because the terminal nodes of the link that connects between clusters are a long way each other on the two-dimensional space (see Fig. 1). However, Lebesgue has simple structure and two dimensional coordinates can be easily converted into the corresponding IDs on it. Therefore, Lebesgue curve is often used for ID conversion in P2P networks, especially for mobile ones where the computing power is limited and it is required to reduce overhead of complex processing. Our space filling curve aims to be able to convert ID as easily as the Lebesgue curve and to give an ID the correlation with the geographical proximity of the node which has the next ID such as Hilbert curve.

3.2 Consideration on the Curves and Proposed Curve

The conventional space filling curves such as Hilbert curve and $\beta\Omega$-indexing have self-similarity because they are constructed by hierarchical processes. But they are not considered to design to hierarchically spread nodes. In the curves, it can be possible to change the hierarchical level with respect to each sub area, for example there are 2^4 nodes to be traveled (the hierarchical level is 2) in an sub area and there are 2^6 nodes to be traveled (the hierarchical level is 3) in another sub area. However, in the sub areas, all nodes should be in the same hierarchical level and they should be relabeled when the number of nodes increased. It is difficult for the labeling scheme of the curves to be assigned hierarchically-spread nodes efficiently. In order to achieve scalability for information storage and retrieval on P2P networks based on the curves, service providers need to adopt a hierarchical structure in an upper service layer. The curve to be proposed can be flexibly assigned and label hierarchically-spread nodes in each area. It can travel all nodes on the curve easily. The curve can handle hierarchical-spread nodes and achieve scalability at the labeling scheme.

Only Lebesgue curve could be assigned such nodes by enhancing the curve to assign hierarchical nodes at the middle point of each edge which goes across an area vertically. In the evaluation section, we compare our curve with the enhanced Lebesgue curve.

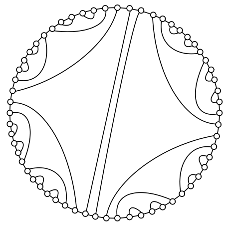

Figure 3: Hierarchical Chordal Ring Network ($N = 62$)

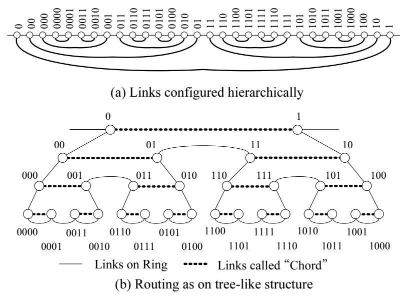

Figure 4: A cluster of HCRN

3.3 Constitution of the Space Filling Curve

In this section, we explain about the proposed space filling curve[8]. The proposed space filling curve is composed based on Hierarchical Chordal Ring Network (HCRN)[7] shown in Fig.3 by enhancing HCRN to two dimensions.

HCRN constructs the tree structure topology on the ring by connecting the link on the node on a cluster edge after clustering by recursive division of node clusters on the ring (Fig.4). Here, each node's ID is expressed by a variable-length gray code according to the hierarchy where the node is located, and each node queues up in the order of the gray code in the hierarchy. The binary number and the gray code can be easily converted by using the following exclusive-OR operation and the simple algorithm. If a certain n digit binary number is assumed to be $\mathbf{b}$ and the gray code corresponding to it is assumed to be $\mathbf{g}$, they are possible to be converted to each other.

Algorithm Binary2Gray

$$\mathbf{g} = \mathbf{b} \oplus (\mathbf{b} >> 1)$$

Algorithm Gray2Binary

1: Given a gray code $\mathbf{g} = (g_1, g_2, \cdots, g_n)$
2: $flag_{rev} \leftarrow$ false
3: **for** $i \leftarrow 1$ **to** n **step** 1
4:   **if** $g_i =$ '1' **then**
5:     **if** $flag_{rev} =$ false
6:     **then** $b_i \leftarrow$ '1' **else** $b_i \leftarrow$ '0' **endif**
7:     $flag_{rev} \leftarrow$ **not** $flag_{rev}$
8:   **else**
9:     **if** $rev_f lag =$ false
10:     **then** $b_i \leftarrow$ '0' **else** $b_i \leftarrow$ '1' **endif**
11: **endfor**
12: **return** the binary code $\mathbf{b} = (b_1, b_2, \cdots, b_n)$

In Lebesgue curve, given a geographical position $(\mathbf{x_i}, \mathbf{y_i})$ of a node n_i ($\mathbf{x_i} = (x_1, x_2, \cdots, x_H), \mathbf{y_i} = (y_1, y_2, \cdots, y_H)$), the node's geographical label $\mathbf{p_i}$ is given as $\mathbf{p_i} = (x_1, y_1, x_2, y_2, \cdots, x_H, y_H)$. Here, H denotes both the length of geographical information and the maximum number of hierarchical level.

In our curve, first, in order to let the curve be a closed curve, we define that a new geographical information of a node n_i is given as $\mathbf{p_i} =$ Binary2Gray$(x_1, y_1, \overline{x_2}, \overline{y_2}, \cdots, x_{2j-1}, y_{2j-1}, \overline{x_{2j}}, \overline{y_{2j}}, \cdots)$. Here, $\overline{x}$ means bit inversion and $j \in \mathbb{N}$. This definition provides a result that the nearer two nodes is located, the smaller the Hamming distance between their node labels is.

Next, to handle hierarchically-spread nodes, we define the following algorithm to label ID to each node. Each node is labeled by this scheme in the chronological order of the nodes.

Given

$\mathbf{p_i} = f(\mathbf{x_i}, \mathbf{y_i})$: The geographical label of each node n_i
H : Maximum hierarchical level

Procedure

1: Given $\mathbf{p_i} = (p_1, p_2, p_3, \cdots, p_{2H})$
2: **for** $j \leftarrow 2$ **to** $2H$ **step** 2
3:   **if** Label $(p_1, \cdots, p_j)$ has not been used yet
4:   **then** Label $\mathbf{l_i} \leftarrow (p_1, \cdots, p_j)$ and **goto** 7.
5:   **endif**
6: **endfor**
7: end.

The enhanced Lebesgue curve is obtained by connecting nodes in the numerical order of the label $\mathbf{l_i}$. Note, however, suppose $(l_1, l_2, \cdots, l_m, 0, 1) < (l_1, l_2, \cdots, l_m) < (l_1, l_2, \cdots, l_m, 1, 0)$ when the length of the labels are different.

The proposed curve is obtained by connecting nodes in the numerical order of Gray2Binary($\mathbf{l_i}$). Note, however, suppose $(l_1, \cdots, l_{m-1}, 0) < (l_1, \cdot, l_{m-1}, 0, l_{m+1}, \cdots, l_n)$ and $(l_1, \cdots, l_{m-1}, 1, l_{m+1}, \cdots, l_n) < (l_1, l_2, \cdots, l_{m-1}, 1)$ when the length of the labels are different.

An example of the proposed curve is shown in Fig. 5. Figure 6 shows the lowest level nodes and their connections out of the curve in order to let it be easier to compare with the conventional space filling curves such as Hilbert curve.

3.4 Applying to Virtual Network Map

3.4.1 Overview

We use the proposed space filling curve to calculate IDs of nodes from the geographical coordinate of each node uniformly and configure P2P network. But, if nodes are unevenly distributed, some problems occur such that the load concentrates on specific nodes, and that the balance of network topology is collapsed and the diameter of network becomes large. Because there is positive correlation in the communication delay in the network and the geographical proximity between arbitrary nodes, LL-net makes nodes a cluster

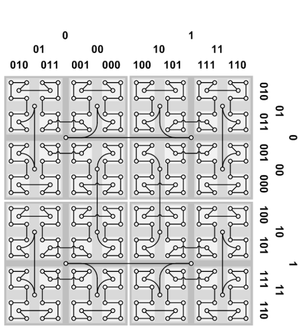

Figure 5: Proposed curve

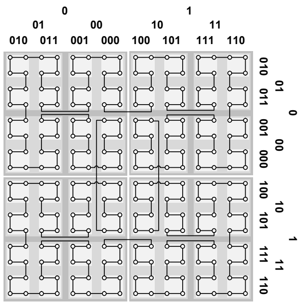

Figure 6: The links of the lowest level in the proposed curve

using geographical information to construct low-delay P2P network. Moreover, it is possible to deal with the problem of the bias of the load of each node and the increase of network diameter, because the multi-level network is dynamically made for geographical uneven distribution of nodes. In the large network that the distances between nodes are hundreds of kilometers-scale, the geographical proximity is thought to have great influence on the communication delay. However, the influence does not grow so much in the network that the distances between nodes are kilometers scale. Rather, the increase of the adjustment cost for the disorder of the hierarchical structure by the uneven distribution of the node becomes the big problem. In the foregoing paragraph, it have been described that the proposed space filling curve can consider the geographical proximity and the proximity in the network between nodes more by almost the same ID conversion cost as the Lebesgue curve. In the network with many mobile terminals, to reduce the ID conversion cost becomes more important than to minimize the real communication delay of the connection link, from the viewpoint of low ability for computing / communication and mobility of the mobile terminals. Therefore we propose the configuration method of the P2P overlay network which lower the communication delay in the network and reduce the problem of uneven distribution of nodes under the supposition that each node can acquire by information about the communication delay.

3.4.2 Construction of Virtual Network Map

At first, in the proposed method, each node to build an overlay network is given a virtual coordinate with a virtual network map. We use Vivaldi[4] as configuration technique of the virtual network map in this method. Hereby, the virtual coordinates are set by considering real delay between nodes and nodes are not concentrated at the specific location on the virtual network map. Next, node ID is assigned to each node in the virtual network map by applying the proposed space filling curve.

In this method, we use Vivaldi to give the coordinate of each node on the virtual network map. Vivaldi is technique to build a coordinate system for autonomous decentralized in each node by coordinating the error of actual delay value with Euclid distance using the principle of the spring gradually. However, there are problems that the convergence of the coordinate system is slow because width of the adjustment of the error is small. Therefore, the prolongment of the settling time becomes the problem especially when the coordinates of the initial solution are quite different from an actual delay value. In our method, the geographical location information are given as initial coordinates. We assume that each node which participate in the network can acquire location information by GPS. Each node uniformly calculates initial coordinate from the acquired location information. Figure 7 shows the outline of Vivaldi.

The P2P overlay network that considers the delay time and the uneven distribution of nodes can be constructed by calculating node ID with the proposed space filling curve for the nodes on the virtual network map with Vivaldi.

4 Numerical Evaluation

The advantage of the proposed method is evaluated by the following points.

- The index that shows the physical neighbor node is also near on space filling curve (Figs. 9 and 10)
- The communication delay which necessary for travelling all over the filling curve (Figs. 11 and 12)

First, we compared the proposed curve with Lebesgue curve and Hilbert curve form the standpoint of the proximity index. Next, we compared it with the enhanced Lebesgue curve. Nodes on the proposed curve and the enhanced Lebesgue curve can belong to various hierarchical levels as shown in Fig. 5 although nodes on Hilbert curve belong to the same hierarchical level.

4.1 Proximity with Neighbor Nodes on the Curves

First of all, we evaluated how far the nodes which is physically in four neighborhoods which are on the filling curves, using logarithmic index range R_{avg}^{curve} shown in [14]. Suppose that each node n_i has a sequence number $seq(i)$ on the curve and it is located on coordinate $pos(i) = (x_i, y_i)$. Proximity index R_{avg}^{curve} can be calculated by the following expressions.

$$
\begin{aligned}
r_1^{curve}(i) &= \log(|seq(i) - seq(pos^{-1}(x-1, y))| + 1) \\
r_2^{curve}(i) &= \log(|seq(i) - seq(pos^{-1}(x+1, y))| + 1) \\
r_3^{curve}(i) &= \log(|seq(i) - seq(pos^{-1}(x, y-1))| + 1) \\
r_4^{curve}(i) &= \log(|seq(i) - seq(pos^{-1}(x, y+1))| + 1) \\
R_{avg}^{curve} &= \text{avg}_i\{r_j^{curve}(i)\}, \quad (j \in \{1, 2, 3, 4\})
\end{aligned}
$$

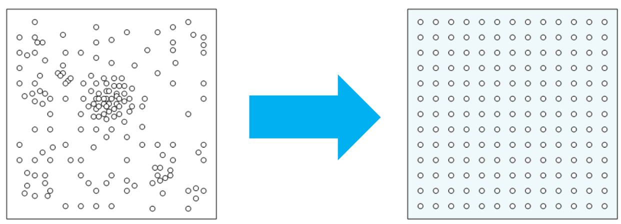

Figure 8: Construction of Virtual Network Map by Vivaldi

- Input
  - N : Number of nodes
  - $Nodes = \{n_i\}(i = \{1, 2, \cdots, N\})$: The set of the nodes
  - $\mathbf{x_i} = (x_i, y_i)$: The geographical position attribute of each node n_i
  - $D = \{d_{ij}\}$: The matrix of delay time d_{ij} between arbitrary nodes n_i and n_j
- Output
  - $\mathbf{p_i} = (p_i, q_i)$: The position of each node n_i on the delay map
- Objective function $cost$
  - To minimize E which is the total sum of the estimation error between communication delay and distance on the delay map

$$\text{minimize } \Sigma_{i,j}(d_{ij} - ||\mathbf{p_i} - \mathbf{p_j}||)^2$$

(a) Formulation

- F : Force vector that the spring between node i and j exerts on node i
- $u(i, j)$: Unit vector which gives the direction of the force between i–j
- t : The time constant to converge the spring model

While $(cost - cost_{prev}) < \epsilon$
  foreach $i \in Node$
    $F := 0$;
    foreach $j \in Node$
      $e := d_{ij} - ||n_i - n_j||$;
      $F := F + e \times u(i, j)$;
    end;
    $\mathbf{p_i} := \mathbf{p_i} + t \times F$
  end;
end.

(b) Algorithm

Figure 7: Overview of Vivaldi

The evaluation results are shown in Fig.9.

A physically neighbour node can be assigned to nearer sequence number on the proposed curve comparing with the Lebesgue curve. It can be a remarkable improvement, especially considering that R^{curve}_{avg} is the log scale. Although Hilbert curve shows much better result, it is difficult to use Hilbert curve on the P2P network because there is no easy way for nodes on Hilbert curve to decide where the next node is.

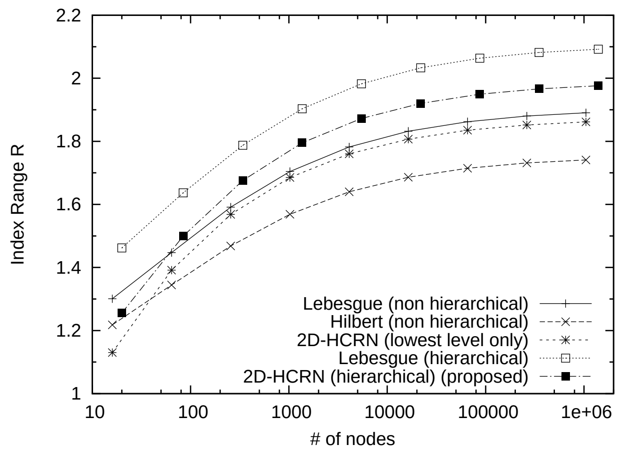

Figure 9: Average of logarithmic index range R^{curve}_{avg}

4.2 Length of Curves

We evaluated the delay for various numbers of nodes with computer simulations. In the simulations, we assume that the number of nodes are changed from 50 to 10,000, and each result is the average of 30 trials.

Figure 10 shows the square sum of delay between each node and each next node on the curves where the geographical position of nodes are generated by the uniform distribution. As a result, the proposed curve has been improved by

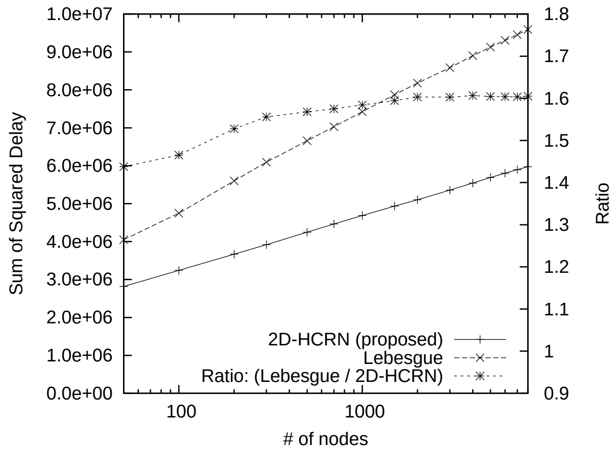

Figure 10: Sum of squared delays between successive two nodes on the curves with randomly distributed nodes

as much as 30-60% against Lebesgue curve. It can be said that the distance between any adjacent nodes on the proposed curve are always small without large bias than the enhanced Lebesgue curve.

4.3 Traversal Time on Virtual Network Map

Finally, we evaluated the traversal time of all nodes on the proposed curve and on Lebesgue curve (Z-ordering) of each when the virtual network map is used and when it is not used. The number of nodes is given between 10 to 100 in this experiment. Figures 11 and 12 show the result of simulations.

On the condition that those curve are not on a virtual network map, there is few difference between the proposed curve and Lebesgue curve because most of the delay is dominated by physical link connectivity of each node rather than the proximity of neighbor nodes on the curve. However, when the virtual network map is used, the traversal time of the proposed curve is smaller than that of Lebesgue curve because the delay between nodes is proportional to the distance between the nodes on the virtual network map. In this way, our proposed curve is effective in the network with delay variations.

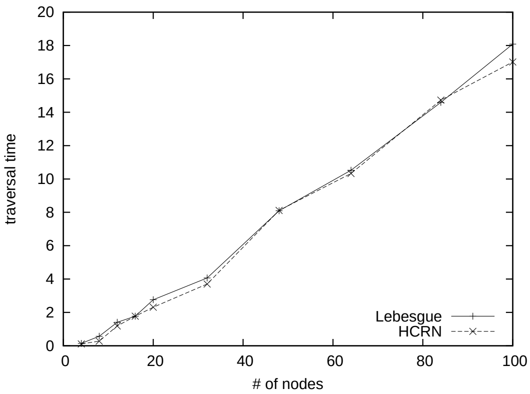

Figure 11: Traversal time on the curves not o virtual network map

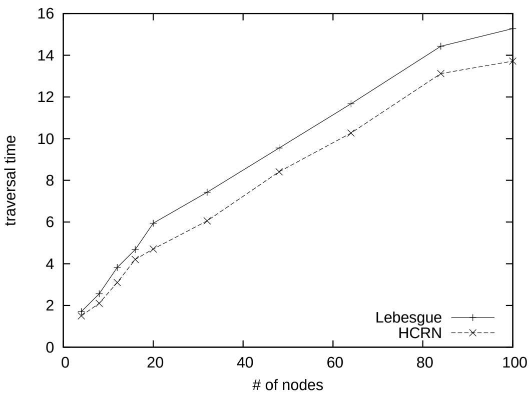

Figure 12: Traversal time on the curves on virtual network map

5 Conclusion

In this paper, we proposed the method which configure small delay structured P2P overlay network by assigning a label considering the geographic information of each terminal node that became peer. The proposed space filling curve to configure the P2P network converts geographic information into node ID (label) of P2P network, and it can search information over specified range efficiently, and can construct hierarchical connection structure for scalability. Through the numerical evaluation, we confirmed that the proposed curve is more suitable for hierarchical-spread nodes than the conventional curves.

For future work, we need the performance evaluation of the proposed space filling curve and the P2P overlay network in the real network environment especially in node distribution with bias, and the reexamination of the routing entry of each node to improve the performance of this method.

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